# MENG610 Advanced Dynamics

## Fundamentals

**Kinematic relationships are used to help us determine the trajectory of a golf ball, the orbital speed of a satellite, and the accelerations during acrobatic flying.**

# MENG610 Advanced Dynamics

## Introduction

- **Dynamics includes:**

***Kinematics***: study of the geometry of motion.
Relates displacement, velocity, acceleration, and time ***without reference*** to the cause of motion.

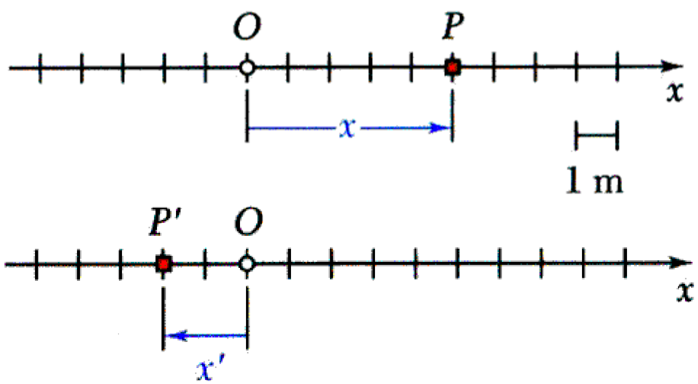

***Kinetics***: study of the relations existing between the forces acting on a body, the mass of the body, and the motion of the body. Kinetics is used to predict the motion caused by given forces or to determine the forces required to produce a given motion.

- **Particle kinetics includes:**

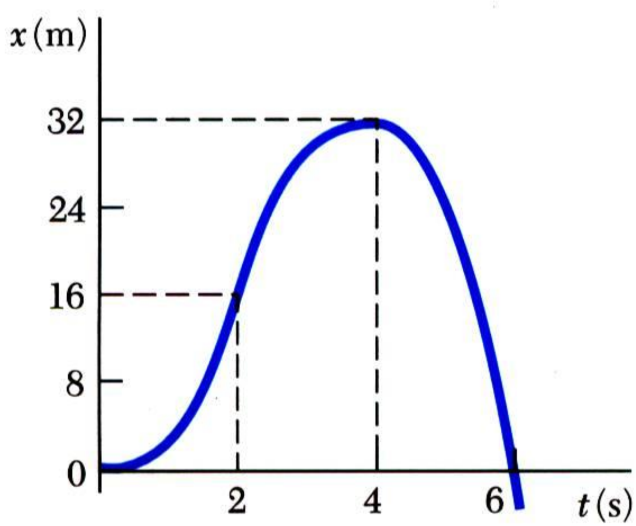
  - ***Rectilinear motion***: position, velocity, and acceleration of a particle as it moves along a straight line.

- ***Curvilinear motion***: position, velocity, and acceleration of a particle as it moves along a curved line in two or three dimensions.

## Rectilinear Motion: Position, Velocity & Acceleration




- ***Rectilinear motion:*** particle moving along a straight line
- ***Position coordinate:*** defined by positive or negative distance from a fixed origin on the line.

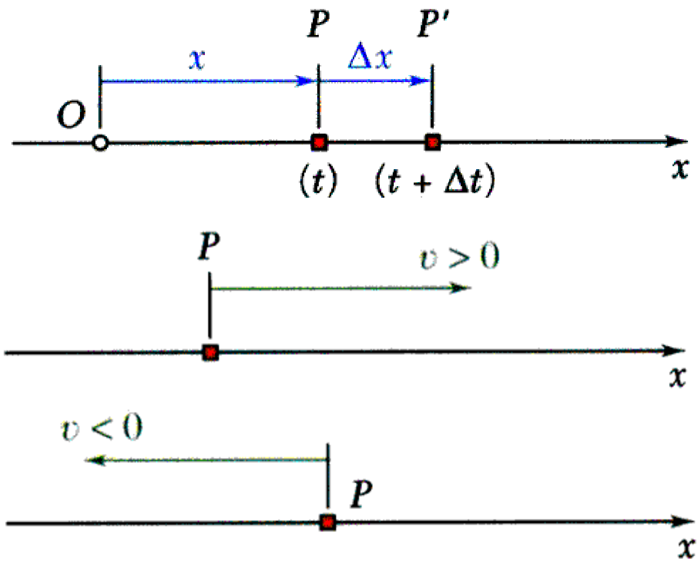


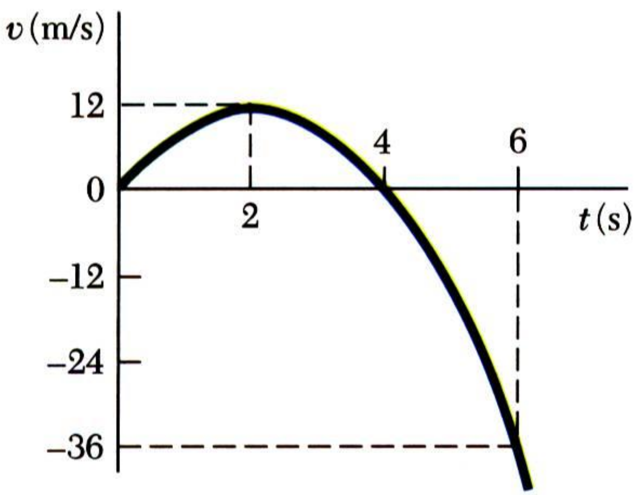

- The ***motion*** of a particle is known if the position coordinate for particle is known for every value of time *t*.
- May be expressed in the form of a function, e.g.,

$$x = 6t^2 - t^3$$

or in the form of a graph *x* vs. *t*.

## Rectilinear Motion: Position, Velocity & Acceleration




- Consider particle which occupies position *P* at time *t* and *P'* at *t*+*Δt*,

$$\textit{Average velocity} = \frac{\Delta x}{\Delta t}$$

$$\textit{Instantaneous velocity} = v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

- Instantaneous velocity may be positive or negative. Magnitude of velocity is referred to as *particle speed*.

- From the definition of a derivative,

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

e.g., $x = 6t^2 - t^3$

$$v = \frac{dx}{dt} = 12t - 3t^2$$

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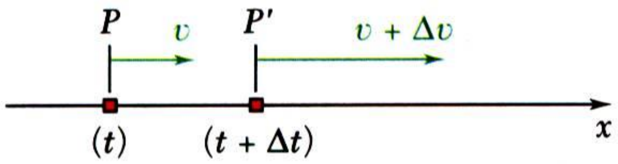
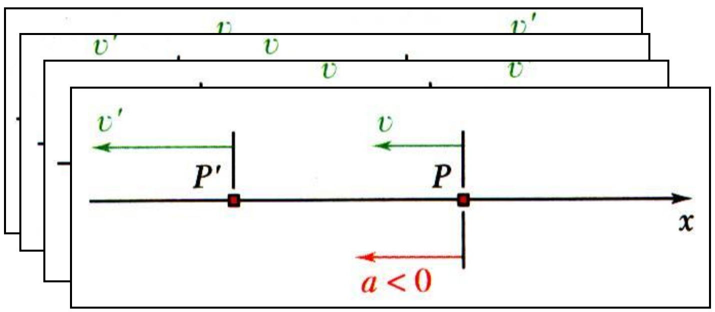
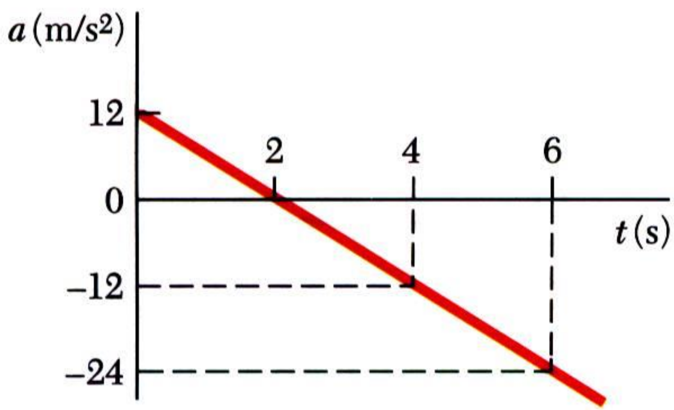
## Rectilinear Motion: Position, Velocity & Acceleration




- Consider particle with velocity $v$ at time $t$ and $v'$ at $t+\Delta t$,

$$\textit{Instantaneous acceleration} = a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

- Instantaneous acceleration may be:
  - positive: increasing positive velocity or decreasing negative velocity
  - negative: decreasing positive velocity or increasing negative velocity.

- From the definition of a derivative,

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 x}{dt^2}$$

e.g. $v = 12t - 3t^2$

$$a = \frac{dv}{dt} = 12 - 6t$$

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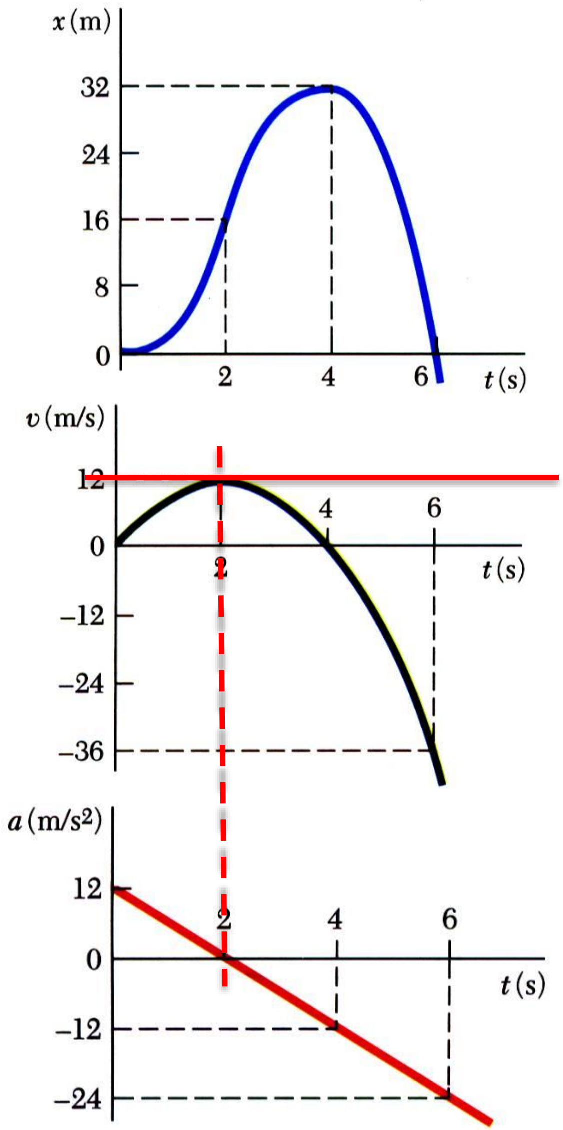
## Rectilinear Motion: Position, Velocity & Acceleration




- From our example,

$$x = 6t^2 - t^3$$

$$v = \frac{dx}{dt} = 12t - 3t^2$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = 12 - 6t$$

- What are $x$, $v$, and $a$ at $t = 2$ s ?

  - at $t = 2$ s, $x = 16$ m, $v = v_{max} = 12$ m/s, $a = 0$

- Note that $v_{max}$ occurs when $a$=0, and that the slope of the velocity curve is zero at this point.
- What are $x$, $v$, and $a$ at $t = 4$ s ?

  - at $t = 4$ s, $x = x_{max} = 32$ m, $v = 0$, $a = -12$ m/s$^2$

## Determination of the Motion of a Particle

- **We often determine accelerations from the forces applied (kinetics will be covered later)**
- **Generally have three classes of motion**
  - acceleration given as a function of *time*, $a = f(t)$
  - acceleration given as a function of *position*, $a = \mathrm{f}(x)$
  - acceleration given as a function of *velocity*, $a = \mathrm{f}(v)$

- **Can you think of a physical example of when force is a function of position? When force is a function of velocity?**

**a spring**

**drag**

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## Acceleration as a function of time, position, or velocity

| If…. | Kinematic relationship | Integrate |
|---|---|---|
| $a = a(t)$ | $\frac{dv}{dt} = a(t)$ | $\int_{v_0}^{v} dv = \int_{0}^{t} a(t)\, dt$ |
| $a = a(x)$ | $dt = \frac{dx}{v}$ and $a = \frac{dv}{dt}$ ↓ $v\, dv = a(x)\, dx$ | $\int_{v_0}^{v} v\, dv = \int_{x_0}^{x} a(x)\, dx$ |
| $a = a(v)$ | $\frac{dv}{dt} = a(v)$ | $\int_{v_0}^{v} \frac{dv}{a(v)} = \int_{0}^{t} dt$ |
| | $v\frac{dv}{dx} = a(v)$ | $\int_{x_0}^{x} dx = \int_{v_0}^{v} \frac{v\, dv}{a(v)}$ |

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## Quiz

**Q1: What is true about the kinematics of a particle?**

a) The velocity of a particle is always positive
b) The velocity of a particle is equal to the slope of the position-time graph
c) If the position of a particle is zero, then the velocity must zero
d) If the velocity of a particle is zero, then its acceleration must be zero

**Q2: Ball tossed with 10 m/s vertical velocity from window 20 m above ground.**

Determine:

- velocity and elevation above ground at time $t$,
- highest elevation reached by ball and corresponding time, and
- time when ball will hit the ground and corresponding velocity.